1. Electrostatics
   a. Charge
   b. Voltage
   c. Current
   d. Resistance

2. Circuit Analysis Basics
   a. Resistor simplification
      i. Parallel
      ii. Series
   b. Source Equivalents
      i. Thevenin
      ii. Norton
   c. Node Analysis
   d. Loop Analysis

3. Transient Circuits
   a. RC Circuits
   b. RL Circuits

4. AC Circuits
   i. RMS
   ii. Phasor Transforms
   iii. AC impedance
   iv. AC Steady State analysis

5. Power
   a. DC Power
      i. Power supplied
      ii. Power Absorbed
   b. AC Power
      i. Complex power
      ii. power factor

6. Transformers
   a. Current and Voltage in an Ideal transformers
   b. Impedance seen at the input of an ideal transformer

7. Operational Amplifiers (OP AMPS)
   a. Ideal OP-AMPS
   b. solving OP-AMP Circuits

8. Resonant Circuits
   a. Series Resonance
   b. Parallel Resonance
   c. Quality Factor
   d. Bandwidth
What you need to know:

1. Electrostatics
   a. Charge
      i. Units: Coulombs (C), 1 C is defined as the charge of $6.24 \times 10^{16}$ electrons. The charge of an electron is $1.6 \times 10^{-19}$ C.
      ii. The force of one charge on another charge: \[
         \vec{F}_{12} = \frac{Q_1 Q_2}{4 \pi \varepsilon_0 r^2} \hat{a}_{12}.
         \]
         Where:
         1. \( Q_i \) is the \( i \)th point charge
         2. \( \vec{F}_{12} \) is the force on charge 2 due to charge 1
         3. \( r \) is the distance between the two charges
         4. \( \hat{a}_{12} \) is a unit vector directed from 1 to 2
         5. \( \varepsilon \) is the permittivity of the medium (how capable is the medium in which the charges exist of allowing these forces to exist)
   
   b. Voltage – The potential difference between two points, is the work done per unit charge required to move the charge between two points.

   c. Current – Rate of charge passing across a surface:
      \[
      I = i(t) = \frac{dQ}{dt}.
      \]

   d. Resistance – Measure of the ability of charge to move from one point to another for a given potential difference (voltage).
      \[
      R = \frac{\rho L}{A}.
      \]
      \[\text{[Diagram: Electrical circuit with voltage V and current I, R is shown.]}\]

   e. Ohm’s Law
      \[ V = IR \text{ or } v(t) = i(t)R \]

2. Circuit Analysis
   a. Resistor Simplification
      i. Two resistors in series: \( R_{TOT} = R_1 + R_2 \)
      
      ii. Two resistors in parallel:
          \[
          R_{TOT} = \frac{R_1 R_2}{R_1 + R_2}.
          \]

   b. Node Analysis – Using Kirchhoff’s Current Law (KCL): \( \Sigma I = 0 \) at any node.
      And:
      \[
      I_{AB} = \frac{V_A - V_B}{R_{AB}}.
      \]

   Write a system of equations to solve for unknown node voltages.

   c. Loop Analysis – Using Kirchhoff’s Voltage Law (KVL): \( \Sigma V = 0 \) around any closed path.
       Write a system of equations to solve for the unknown currents in each branch.

   d. Source equivalents – At any port a linear circuit can be simplified into an ideal source and a resistance
      i. Thevenin Equivalent Circuit – circuit is simplified into a series combination of an ideal voltage source and a resistance.
         1. Find the open circuit voltage \( V_{oc} \) at the port of interest
2. Find the equivalent resistance \((R_{TH})\) at the port of interest
   Or Find the short circuit current \((I_{SC})\) at the port of interest
   \[
   R_{TH} = \frac{V_{oc}}{I_{sc}}
   \]

ii. Norton Equivalent Circuit – Circuit is simplified into a parallel
combination of an ideal current source and a resistance
   1. Find the short circuit current \((I_{SC})\) at the port of interest
   2. Find the equivalent resistance \((R_{TH})\) at the port of interest
       Or Find the open circuit voltage \((V_{oc})\) at the port if interest
       \[
       R_{TH} = \frac{V_{oc}}{I_{sc}}
       \]

3. Transient Circuits – Circuits that contain capacitors and/or inductors as well as resistors.
   Voltages or Current sources are switched on or off at \(t = 0\). Response is analyzed
   a. Capacitors: \(C = \frac{εA}{d}\); capacitance for a parallel plate capacitor
   \[
   i(t) = C \frac{dV_c(t)}{dt}
   \]
   b. Inductors: \(L = \frac{NΦ}{i(t)}\); inductance for a coil with \(N\) turns and magnetic flux \(Φ\) enclosed in the coil
   \[
   v(t) = L \frac{di_c(t)}{dt}
   \]
   Requires solving first order (C or L only) or second order (L and C) differential eqns.

4. AC Circuits – Steady state analysis of circuits with (periodic) sinusoidal voltage and
current sources.
   a. Capacitors replaced with frequency dependent impedance \(Z_C = \frac{1}{jωC}\)
   b. Inductors replaced with frequency dependent impedance \(Z_L = jωL\)
   c. Circuit Analysis is simplified from DIFEQ’s to (complex) algebraic techniques
      used in resistive circuit analysis

5. Power
   a. DC (Resistive) Circuits
      i. Real power \(P\) supplied by a source; \(P = VI\)
      ii. Power absorbed by a resistor; \(P = \frac{V^2}{R} = I^2R\)
      iii. Circuits with multiple sources may have some sources ABSORBING
           power. (Battery charging)
   b. AC (Complex) Circuits
      Complex power, \(S = P + jQ\)
      \[
      \text{Real Power } P = (\frac{1}{2})V_{max}I_{max}\cosθ = V_{rms}I_{rms}\cosθ
      \]
      \[
      \text{Complex Power } Q = (\frac{1}{2})V_{max}I_{max}\sinθ
      \]
\[ \theta \text{ is the angle measured from } V \text{ to } I. \]

6. Transformers: Two coils in proximity sharing magnetic flux.

\[ n = \frac{N_1}{N_2} \text{ ; the ratio of the number of turns in the two coils } \]

\[ n = \frac{|V_1|}{|V_2|} = \frac{|I_2|}{|I_1|} \]

Beware of the Dots!

7. Operational Amplifiers

For Ideal Op-Amps:
- No current flows into the input terminals
- The input terminals have the same voltage

8. Resonant Circuits – Parallel and Series LC circuits have a bandpass frequency response. This response is located at a center frequency \( (\omega_o) \) and has a bandwidth (BW)

\[ \text{Resonant Frequency: } \omega_o = \frac{1}{\sqrt{LC}} \]

\[ \text{Impedance at Resonance: } Z = R \]

\[ Z_L + Z_C = 0 \]

\[ Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC} \]

\[ BW = \frac{\omega_o}{Q} \]

\[ Q = \frac{\omega_o RC}{\omega_o L} \]

\[ V_L \text{ as a } fcn \text{ of } \omega \]

\[ V_L \text{ @ DC: } V_L = \mathbf{I} \mathbf{Z}_L \]

\[ V_L(\omega) = \mathbf{I} j\omega L = 0 \]

\[ \omega = 0 \]

\[ V_L\text{ - MAX} \]

\[ 0.7V_{\text{MAX}} \]

\[ 0.7V_{\text{MAX}} \cdot 0.7 \]

\[ \text{Bandwidth} \]

\[ \omega_L \text{ } \omega_0 \text{ } \omega_H \]

\[ \text{Quality Factor} \]

\[ \text{Parallel Resonance: } \infty = \mathbf{Z}_L \parallel \mathbf{Z}_C \]

\[ \text{Series Resonance: } Q = \omega_o RC \]

\[ Q = \omega_o RC = \frac{R}{\omega_o L} \]

\[ V_{\text{L - MAX}} \]

\[ 0.7V_{\text{MAX}} \cdot 0.7 \]

\[ \text{Bandwidth} \]
ELECTRIC CIRCUITS

UNITS
The basic electrical units are coulombs for charge, volts for
current, amperes for current, and ohms for resistance and
impedance.

ELECTROSTATICS
\[ F_i = \frac{Q_i Q_j}{4\pi \varepsilon_0 r^2} a_{i,12}, \text{ where} \]
\[ F_i = \text{the force on charge } 2 \text{ due to charge } 1, \]
\[ Q_i = \text{the } i\text{th point charge}, \]
\[ r = \text{the distance between charges } 1 \text{ and } 2, \]
\[ a_{i,12} = \text{a unit vector directed from } 1 \text{ to } 2, \text{ and} \]
\[ \varepsilon = \text{the permittivity of the medium}. \]
For free space or air:
\[ \varepsilon = \varepsilon_0 = 8.85 \times 10^{-12} \text{ Farads/meter} \]

Electrostatic Fields
Electric field intensity \( E \) (volts/meter) at point 2 due to a
point charge \( Q_i \) at point 1 is
\[ E = \frac{Q_i}{4\pi \varepsilon_0 r^2} a_{1,2} \]
For a line charge of density \( \rho \text{ C/m} \) on the z-axis, the radial
electric field is
\[ E_z = \frac{\rho}{2\pi \varepsilon_0} a_z. \]
For a sheet charge of density \( \rho \text{ C/m}^2 \) in the x-y plane:
\[ E_x = \frac{\rho}{2\pi \varepsilon_0}, z > 0 \]
Gauss’ law states that the integral of the electric flux density
\( D = \varepsilon E \) over a closed surface is equal to the charge enclosed
or
\[ \oint_S D \cdot dS = Q \]
The force on a point charge \( Q \) in an electric field with
intensity \( E \) is \( F = QE \).
The work done by an external agent in moving a charge \( Q \) in
an electric field from point \( p_1 \) to point \( p_2 \) is
\[ W = \int_{p_1}^{p_2} \mathbf{E} \cdot d\mathbf{l} \]
The energy stored \( W_E \) in an electric field \( E \) is
\[ W_E = \frac{1}{2} \int \varepsilon |E|^2 \, dV \]
Voltage
The potential difference \( V \) between two points is the work
per unit charge required to move the charge between the
points.
For two parallel plates with potential difference \( V \), separated
by distance \( d \), the strength of the E field between the plates is
\[ E = \frac{V}{d} \]
directed from the + plate to the – plate.

Current
Electric current \( i(t) \) through a surface is defined as the rate of
charge transport through that surface or
\[ i(t) = \frac{dq(t)}{dt}, \text{ which is a function of time } t \]
since \( q(t) \) denotes instantaneous charge.
A constant current \( i(t) \) is written as \( i \), and the vector current
density in amperes/m^2 is defined as \( J \).

Magnetic Fields
For a current carrying wire on the x-axis
\[ \mathbf{H} = \frac{I}{\mu} \frac{a_0}{2\pi r}, \text{ where} \]
\[ \mathbf{H} = \text{the magnetic field strength (amperes/meter)} \]
\[ B = \text{the magnetic flux density (tesla)} \]
\[ a_0 = \text{the unit vector in positive } \phi \text{ direction in cylindrical}
coordinates,} \]
\[ I = \text{the current, and} \]
\[ \mu = \text{the permeability of the medium}. \]
For air: \( \mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)
Force on a current carrying conductor in a uniform magnetic
field is
\[ \mathbf{F} = I \mathbf{L} \times \mathbf{B}, \text{ where} \]
\[ \mathbf{L} = \text{the length vector of a conductor}. \]
The energy stored \( W_H \) in a magnetic field \( H \) is
\[ W_H = \frac{1}{2} \int_{V} \mathbf{H} \cdot |\mathbf{H}|^2 \, dV \]

Induced Voltage
Faraday's Law: For a coil of \( N \) turns enclosing flux \( \phi \):
\[ v = -N \frac{d\phi}{dt}, \text{ where} \]
\[ v = \text{the induced voltage, and} \]
\[ \phi = \text{the flux (webers) enclosed by the } N \text{ conductor turns; and} \]
\[ \phi = \int_a B \cdot dS \]

Resistivity
For a conductor of length \( L \), electrical resistivity \( \rho \), and area
\( A \), the resistance is
\[ R = \frac{\rho L}{A} \]
For metallic conductors, the resistivity and resistance vary
linearly with changes in temperature according to the
following relationships:
\[ \rho = \rho_0 [1 + \alpha (T - T_0)], \text{ and} \]
\[ R = R_0 [1 + \alpha (T - T_0)], \text{ where} \]
\( \rho_0 \) is resistivity at \( T_0 \), \( R_0 \) is the resistance at \( T_0 \), and
\( \alpha \) is the temperature coefficient.
Ohm's Law: \( V = IR; v(t) = i(t) R \)
Resistors in Series and Parallel

For series connections, the current in all resistors is the same and the equivalent resistance for \( n \) resistors in series is

\[
R_T = R_1 + R_2 + \ldots + R_n
\]

For parallel connections of resistors, the voltage drop across each resistor is the same and the resistance for \( n \) resistors in parallel is

\[
R_T = \frac{1}{1/R_1 + 1/R_2 + \ldots + 1/R_n}
\]

For two resistors \( R_1 \) and \( R_2 \) in parallel

\[
R_T = \frac{R_1 R_2}{R_1 + R_2}
\]

Power in a Resistive Element

\[
P = VI = \frac{V^2}{R} = I^2 R
\]

Kirchhoff's Laws

Kirchhoff's voltage law for a closed path is expressed by

\[
\sum V_{\text{rise}} = \sum V_{\text{drop}}
\]

Kirchhoff's current law for a closed surface is

\[
\sum I_{\text{in}} = \sum I_{\text{out}}
\]

SOURCE EQUIVALENTS

For an arbitrary circuit

The Thévenin equivalent is

\[
V_{oc} \quad a \quad R_{eq} \quad b
\]

The open circuit voltage \( V_{oc} \) is \( V_a - V_b \), and the short circuit current is \( I_{sc} \) from \( a \) to \( b \).

The Norton equivalent circuit is

\[
I_{sc} \quad a \quad R_{eq} \quad b
\]

where \( I_{sc} \) and \( R_{eq} \) are defined above.

A load resistor \( R_L \) connected across terminals \( a \) and \( b \) will draw maximum power when \( R_L = R_{eq} \).

CAPACITORS AND INDUCTORS

\[
C_{eq} = \frac{1}{C_1 + \frac{1}{C_2 + \ldots + \frac{1}{C_n}}} \quad \text{Capacitors in Parallel}
\]

\[
L_{eq} = \frac{1}{L_1 + \frac{1}{L_2 + \ldots + \frac{1}{L_n}}} \quad \text{Inductors in Parallel}
\]

The charge \( q_C(t) \) and voltage \( v_C(t) \) relationship for a capacitor \( C \) in farads is

\[
C = q_C(t)/v_C(t) \quad \text{or} \quad q_C(t) = C v_C(t)
\]

A parallel plate capacitor of area \( A \) with plates separated a distance \( d \) by an insulator with a permittivity \( \varepsilon \) has a capacitance

\[
C = \frac{\varepsilon A}{d}
\]

The current-voltage relationships for a capacitor are

\[
v_C(t) = v_C(0) + \int_0^t \frac{1}{C} i_C(\tau) d\tau
\]

and

\[
i_C(t) = C \frac{dv_C}{dt}
\]

The energy stored in a capacitor is expressed in joules and given by

\[
\text{Energy} = C v_C^2 / 2 = q_C^2 / 2C = q_C v_C / 2
\]

The inductance \( L \) of a coil is

\[
L = N^2 / i_L
\]

and using Faraday's law, the voltage-current relations for an inductor are

\[
v_L(t) = L \frac{di_L}{dt}
\]

\[
i_L(t) = i_L(0) + \frac{1}{L} \int_0^t v_L(\tau) d\tau , \text{ where}
\]

\[
v_L = \text{inductor voltage},
\]

\[
L = \text{inductance (henrys), and}
\]

\[
i = \text{current (amperes)}.
\]

The energy stored in an inductor is expressed in joules and given by

\[
\text{Energy} = L i_L^2 / 2
\]

Capacitors and Inductors in Parallel and Series

Capacitors in Parallel

\[
C_{eq} = C_1 + C_2 + \ldots + C_n
\]

Capacitors in Series

\[
C_{eq} = \frac{1}{1/C_1 + \frac{1}{C_2 + \ldots + \frac{1}{C_n}}} \quad \text{Inductors in Parallel}
\]

\[
L_{eq} = \frac{1}{L_1 + \frac{1}{L_2 + \ldots + \frac{1}{L_n}}} \quad \text{Inductors in Series}
\]

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RC AND RL TRANSIENTS

\[ t \geq 0; \quad v(t) = v_c(0)e^{-t RC} + V(1 - e^{-t RC}) \]
\[ i(t) = \left( \frac{V - v_c(0)}{R} \right) e^{-t RC} \]
\[ v_R(t) = i(t) R = \left( V - v_c(0) \right) e^{-t RC} \]
\[ v_L(t) = i(t) L = i(0)e^{-t RL} + V \left( 1 - e^{-t RL} \right) \]
\[ v_c(t) = i(t) R = i(0)e^{-t RL} + V \left( 1 - e^{-t RL} \right) \]
\[ v_L(t) = L \frac{d}{dt} i(t) = -i(t) R e^{-t RL} + V e^{-t RL} \]

where \( v(0) \) and \( i(0) \) denote the initial conditions and the parameters \( RC \) and \( LR \) are termed the respective circuit time constants.

OPERATIONAL AMPLIFIERS

\[ v_o = A(v_1 - v_2) \]

where \( A \) is large (> 10^4), and
\[ v_1 - v_2 \] is small enough as not to saturate the amplifier.

For the ideal operational amplifier, assume that the input currents are zero and that the gain \( A \) is infinite so when operating linearly \( v_2 - v_1 = 0 \).

For the two-source configuration with an ideal operational amplifier,

If \( v_a = 0 \), we have a non-inverting amplifier with
\[ v_o = \frac{R_2}{R_1} v_b + \left( 1 + \frac{R_2}{R_1} \right) v_1 \]

If \( v_b = 0 \), we have an inverting amplifier with
\[ v_o = \frac{R_2}{R_1} v_a \]

AC CIRCUITS

For a sinusoidal voltage or current of frequency \( f \) (Hz) and period \( T \) (seconds),
\[ f = \frac{1}{T} = \omega (2\pi), \text{where} \]
\[ \omega = \text{the angular frequency in radians/s.} \]

Average Value

For a periodic waveform (either voltage or current) with period \( T \),
\[ X_{ave} = \left( \frac{1}{T} \right) \int_0^T x(t) \, dt \]

The average value of a full-wave rectified sine wave is
\[ X_{ave} = \frac{2X_{max}}{\pi} \]
and half this for a half-wave rectification, where \( X_{max} \) is the peak amplitude of the waveform.

Effective or RMS Values

For a periodic waveform with period \( T \), the rms or effective value is
\[ X_{rms} = \sqrt{\left( \frac{1}{T} \int_0^T x(t)^2 \, dt \right)} \]
For a sinusoidal waveform and full-wave rectified sine wave,
\[ X_{rms} = \frac{X_{max}}{\sqrt{2}} \]
For a half-wave rectified sine wave,
\[ X_{rms} = \frac{X_{max}}{2} \]

Sine-Cosine Relations

\[ \cos(\omega t) = \sin(\omega t + \pi/2) = -\sin(\omega t - \pi/2) \]
\[ \sin(\omega t) = \cos(\omega t - \pi/2) = -\cos(\omega t + \pi/2) \]

Phasor Transforms of Sinusoids

\[ P[V_{max} \cos(\omega t + \phi)] = V_{rms} \leq \phi = V \]
\[ P[I_{max} \cos(\omega t + \theta)] = I_{rms} \leq \theta = I \]

For a circuit element, the impedance is defined as the ratio of phasor voltage to phasor current.
\[ Z = \frac{V}{I} \]

For a Resistor,
\[ Z_R = R \]

For a Capacitor,
\[ Z_C = \frac{1}{j\omega C} = jX_C \]

For an Inductor,
\[ Z_L = j\omega L = jX_L, \text{where} \]
\[ X_C \text{ and } X_L \text{ are the capacitive and inductive reactances} \]
\[ \text{respectively defined as} \]
\[ X_C = -\frac{1}{\omega C} \text{ and } X_L = \omega L \]

Impedances in series combine additively while those in parallel combine according to the reciprocal rule just as in the case of resistors.
Complex Power
Real power $P$ (watts) is defined by

$$P = (\frac{1}{2})V_{\text{rms}}I_{\text{rms}} \cos \theta$$

$$= V_{\text{rms}}I_{\text{rms}} \cos \theta$$

where $\theta$ is the angle measured from $V$ to $I$. If $I$ leads (lags) $V$, then the power factor ($pf$),

$$pf = \cos \theta$$

is said to be a leading (lagging) $pf$.
Reactive power $Q$ (vars) is defined by

$$Q = (\frac{1}{2})V_{\text{rms}}I_{\text{rms}} \sin \theta$$

$$= V_{\text{rms}}I_{\text{rms}} \sin \theta$$

Complex power $S$ (volt-amperes) is defined by

$$S = V* P + jQ,$$

where $V*$ is the complex conjugate of the phasor current.
For resistors, $\theta = 0$, so the real power is

$$P = V_{\text{rms}}I_{\text{rms}} = V_{\text{rms}}^2/R = I_{\text{rms}}^2 R$$

RESONANCE
The radian resonant frequency for both parallel and series resonance situations is

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 \text{ (rad/s)}$$

Series Resonance

$$\omega_s = \frac{1}{\omega_0 C}\text{ and } Z = R \text{ at resonance.}$$

$$Q = \frac{\omega_s L}{R} = \frac{1}{\omega_0 C R}$$

$$BW = \omega_0 Q \text{ (rad/s)}$$

Parallel Resonance

$$\omega_s = \frac{1}{\omega_0 C}\text{ and } Z = R \text{ at resonance.}$$

$$Q = \omega_s RC = \frac{R}{\omega_0 L}$$

$$BW = \omega_0 Q \text{ (rad/s)}$$

TRANSFORMERS

ELECTRIC CIRCUITS (continued)

$Z_p \rightarrow$

$\begin{array}{c}
N_1 \\
+ \\
V_p \\
- \\
N_2 \\
+ \\
V_s \\
- \\
Z_s
\end{array}$

$$Z_p = a^2 Z_s$$

Turns Ratio

$$a = \frac{N_1}{N_2}$$

$$a = \frac{V_p}{V_s} = \frac{I_p}{I_s}$$

The impedance seen at the input is

$$Z_p = a^2 Z_s$$

ALGEBRA OF COMPLEX NUMBERS
Complex numbers may be designated in rectangular form or polar form. In rectangular form, a complex number is written in terms of its real and imaginary components.

$$z = a + jb \text{, where}$$

$a = \text{the real component,}$

$b = \text{the imaginary component, and}$

$j = \sqrt{-1}$

In polar form

$$z = c \angle \theta \text{, where}$$

$$c = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1} (b/a)$$

$a = c \cos \theta$, and

$b = c \sin \theta$.

Complex numbers are added and subtracted in rectangular form. If

$$z_1 = a_1 + jb_1 = c_1 \cos \theta_1 + jsin \theta_1$$

$$= c_1 \angle \theta_1 \text{ and}$$

$$z_2 = a_2 + jb_2 = c_2 \cos \theta_2 + jsin \theta_2$$

$$= c_2 \angle \theta_2 \text{, then}$$

$$z_1 + z_2 = (a_1 + a_2) + j(b_1 + b_2) \text{ and}$$

$$z_1 - z_2 = (a_1 - a_2) + j(b_1 - b_2) \text{ and}$$

While complex numbers can be multiplied or divided in rectangular form, it is more convenient to perform these operations in polar form.

$$z_1 \times z_2 = (c_1 \times c_2) \angle \theta_1 + \theta_2$$

$$z_1 / z_2 = (c_1 / c_2) \angle \theta_1 - \theta_2$$

The complex conjugate of a complex number $z_1 = (a_1 + jb_1)$ is defined as $z_1^* = (a_1 - jb_1)$. The product of a complex number and its complex conjugate is $z_1 z_1^* = a_1^2 + b_1^2$. 

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ELECTROSTATICS

CHARGE: UNIT IS COULOMB
- 1 electron has a charge of $1.6 \times 10^{-19}$ C
- 1 Coulomb is the charge of $6.24 \times 10^{18}$ e

FORCE: A FORCE OF ONE CHARGE ON ANOTHER

$$\vec{F}_{12} = \frac{q_1 q_2 \hat{a}_{12}}{4 \pi \varepsilon_0 d^2}$$

$q_i$: $i^{th}$ POINT CHARGE COULOMBS

$\varepsilon_0$: PERMITIVITY OF THE MEDIUM (HOW CAPABLE IS THE MATERIAL WHERE THE CHARGE EXISTS TO ALLOW THE FORCES TO EXIST)

FREE SPACE VS. TEFLOM, METALS

d: distance between two charges $q_1$, $q_2$

$\hat{a}_{12}$: UNIT VECTOR FROM $q_1$ TO $q_2$

ELECTRIC FIELD

$$\vec{E}_{12} = q_1 \vec{E}$$
$$\vec{E}_2 = \frac{q_2}{4 \pi \varepsilon_0 r^2} \hat{r}$$

$\hat{r}$: UNIT VECTOR POINTING FROM THE CHARGE TO THE ELECTRIC FIELD EVALUATION POINT

$\vec{E}_a$: ELECTRIC FIELD AT EVALUATION POINT

$$\vec{E}_a = \frac{q}{4 \pi \varepsilon_0 d^2} \hat{r}_a$$

CHARGE
**ANALOGY OF FLUID FLOW AND CURRENT FLOW**

**WATER FLOW**

- Water exits pump at higher pressure $P_a$
- Water enters pump at lower pressure $P_b$

**Flow Rate**

- Flow rate into turbine = flow rate out
- Pressure into turbine > pressure out

**FLOW RATE ANALOGY TO CURRENT FLOW**

- Flow rate: 2 gal/sec
- Current: 2amps

**KVL**

For a closed path

\[ \Sigma V_{risers} = \Sigma V_{drops} \]

**KCL**

\[ \Sigma i = 0 \]

\[ I_{in} = I_{out} \]
VOLTAGE: POTENTIAL DIFFERENCE BETWEEN TWO POINTS

WORK DONE PER UNIT CHARGE REQ'D TO MOVE CHARGE BETWEEN TWO POINTS

POINT A

\[ V_A - V_B = \int_a^b \mathbf{E} \cdot \mathbf{dl} \]

LINE INTEGRAL OF \( \mathbf{E} \)

CURRENT: RATE OF CHANGE PASSING ACROSS A SURFACE

\[ I = i(t) = \frac{dl}{dt} \]

RESISTANCE: MEASURES ABILITY OF CHARGE TO MOVE FROM ONE POINT TO ANOTHER

\[ R = \frac{\rho L}{A} \]

\( \rho \): MATERIAL RESISTIVITY

\( \sigma = \frac{1}{\rho} \): CONDUCTIVITY

OHMS LAW

DC

\[ V = IR \]

AC

\[ V(t) = i(t)R \]

\[ I + V = \frac{V}{R} \]

\[ V_R = V_A - V_B \] "VOLTAGE DROP ACROSS R."

PASSIVE SIGN CONVENTION: POSITIVE CHARGE FLOWS FROM HIGH VOLTAGE TO LOW.
SIMILAR TO A MECHANICAL SYSTEM WE SET UP A COORDINATE SYSTEM THAT WILL ESTABLISH THE POSITIVE DIRECTION.

AFTER CALCULATING VELOCITY (FOR EXAMPLE) IF THE ANSWER IS A POSITIVE NUMBER, OUR ARBITRARY SELECTION OF THE POSITIVE DIRECTION WERE CORRECT.

IF THE ANSWER WAS NEGATIVE, THIS MEANS MOVEMENT IS ACTUALLY IN THE OPPOSITE DIRECTION.

FOR ELECTRIC CIRCUITS WE WILL ALSO ASSIGN AN ARBITRARY DIRECTION OF CURRENT FLOW AND VOLTAGE DIFFERENCE (POLARITY).

**Circuit 1**

<table>
<thead>
<tr>
<th>A</th>
<th>+</th>
</tr>
</thead>
<tbody>
<tr>
<td>V₁ = 2V</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>-</td>
</tr>
</tbody>
</table>

**Terminal A is 2V higher than terminal B**

**Circuit 2**

<table>
<thead>
<tr>
<th>+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vᵢ = -2V</td>
</tr>
<tr>
<td>-</td>
</tr>
</tbody>
</table>

**Terminal B is 2V higher than terminal A**

**Circuit 3**

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>---</td>
</tr>
<tr>
<td>Vᵢ = 2V</td>
</tr>
<tr>
<td>+</td>
</tr>
</tbody>
</table>

**Terminal B is 2V higher than terminal A**

POWER DELIVERED VS. POWER ABSORBED

**Circuit Diagram**

BATTERY DELIVERS POWER TO BULB

BULB ABSORBS POWER FROM BATTERY
Conventions for Current

Positive current is the flow of positive charge (opposite of electron flow).

Negative current is the opposite flow of positive charge.

\[ A \rightarrow I, \quad B \]
\[ A \leftarrow -I, \quad B \]

Both branches have current flow node A to B.

Conventions for Voltage

Voltage between 2 points is the difference in the energy level of a unit charge located at each of the two points.

\[ A + V_{AB} \quad -B \]

Voltage between node A and B is \( V_{AB} = -V_{BA} \).

Energy at node A is greater than that at node B.

If \( V_{AB} = 5 \) volt, voltage at node A is 5 volts more than 5 volts less than.

Voltage at ground always = 0.

Conventions for Power

Power is absorbed or supplied.

If positive current flows into a higher voltage node: power is absorbed.

\[ V = I \cdot R \]

If positive current flows out of a higher voltage node: power is supplied.
Find power absorbed/supplied by each element.

\[ \text{Power supplied} = -VI \]
\[ \text{Power absorbed} = +VI \]

(CHAIRNG BATTERY)

\[ \text{Power supplied} = +VI \]
\[ \text{Power absorbed} = -VI \]

\[ \text{Power absorbed} = -VI \]

\[ R \]
\[ +V - \]

\[ \rho = VI \]
\[ \rho_s = -VI \]
**Power Absorbed**

\[ P = VI = u(t)i(t) \]

**Units:** Watts

**Passive Device Absorbs "Positive" Power Equal to Current Through Times Voltage Across.**

**DC** Direct \( V \), \( I \) Constant

**AC** Alternating \( u(t) \), \( i(t) \) Time Varying

\[ I + V \rightarrow \]

\[ i(t) + u(t) \rightarrow \]

**Ex**

\[ V = 10 \text{ V} \quad I = 5 \text{ Amps} \]

**Find** \( R \), Power Absorbed.

**Soln** \( V = IR \)

\[ R = \frac{V}{I} = 2 \text{ Ohms} \]

\[ P = VI = 50 \text{ Watts} \]

**Ex**

\[ u(t) = 10 \cos(t) \]

**Find** \( i(t) \), \( P(t) \)

**Soln** \( u(t) = i(t)R \)

\[ i(t) = \frac{u(t)}{R} = 5 \cos(t) \text{ Amps} \]

\[ P(t) = 10 \cos(t) \times 5 \cos(t) \]

\[ = 50 \cos^2(t) = 50 \left[ \frac{1}{2} + \frac{1}{2} \cos(2t) \right] \]

\[ = 25 + 25 \cos 2t \text{ Watts} \]

**Instantaneous Average Power**

\[ P_{\text{ave}} = \frac{VI}{2} \quad \text{AC Average Power} \]
More complicated circuits are not as easy to predict before performing circuit analysis.

Once voltages and currents are calculated, we can determine power delivered and absorbed.

**Circuit Elements**

**Sources:**
- **Independent Voltage Source**
  \[ V(t) \]
- **Independent Current Source**
  \[ I(t) \]
- **Dependent Sources**
  - Current
    \[ i = \beta i_0 \]
  - Voltage
    \[ v = a v_0 \]

**Power Absorbed** (Bulb)

**Power Supplied** (Battery)
Simplifying Resistor Circuits.

- Resistor Simplification
- Thevenin Equivalent Circuits
- Norton Equivalent Circuits.

Resistors in Series

\[ R_T = R_1 + R_2 \]

\[ R = R_1 + R_2 \]

Resistors in Parallel

\[ R_T = \frac{(R_1)(R_2)}{R_1 + R_2} \]

\[ R_T = \frac{\rho L}{2A} = \frac{R_1}{2} = \frac{R_2}{2} \]
FIND $I$ IN THIS CKT.

CAN USE LOOP EQUATIONS OR RESISTIVE SIMPLIFICATION

CURRENT FROM SOURCE $I_s$ DELIVERED TO RESISTIVE CIRCUIT.

$4\,\|\,4\,\|\,4\,\|\,4 = \frac{16}{8} = 2\,\Omega$

$V = IR$

$I = 2\,\text{A}$
THEVENIN AND NORTON EQUIVALENT CIRCUITS

AT ANY "PORT" OF A LINEAR CIRCUIT WE CAN REPLACE WITH AN IDEAL SOURCE AND A RESISTANCE.

\[ V_{oc} = V_{th} \]
\[ I_{sc} = I_N \]
\[ R_{th} = \frac{V_{sc}}{I_N} \]
And single resistance

Replace all DC elements with Ideal Source

At any point of a linear circuit, we can

Equivalence Circuits

Knots

Heveling
At node C

\[ \sum I = 0 \]
\[ I_{bc} + I_{ac} + I_1 + I_{cn} = 0 \]

All currents are pointing into node C.

One or more must be negative for KCL to hold.

* If a current or voltage are found to be negative, then actual \( \{V, I\} \) is opposite of original arbitrary assignment.

\[ \text{Find } I_1, I_2, V_B, V_A, V_{R2} \]
\[ V_s = 5 \text{ V} \quad I_0 = 1 \text{ A} \]

\[ R_1 = 6 \text{ } \Omega \quad R_2 = 4 \text{ } \Omega \]

\[ \text{KCL at } A \]
\[ I_1 + I_3 = I_0 = 1 \text{ A} \]
\[ I_1 = 1 - I_3 \]

\[ 5 - V_{R1} - V_{R2} = 0 \]
\[ 5 - I_1 R_1 + I_3 R_2 = 0 \]

\[ V_{R2} = -I_3 R_2 \]

Note passive sign convention

\[ 5 - (1 - I_3) R_1 + I_3 R_2 = 0 \]
\[ 5 - I_1 R_1 + R_1 I_3 + R_2 I_3 = 0 \]
\[ 10 I_3 = 1 \]
\[ I_3 = 0.1 \text{ A} \]
\[ I_1 = 0.9 \text{ A} \]
\[ V_B = -0.4 \text{ V} \quad V_A = -0.4 \]
EX II

\[ V_s \]

[Image of a circuit diagram]

Given
\[ V_s = 5 \text{ V} \]
\[ R_1 = 10 \Omega \]
\[ I_0 = 1 \text{ A} \]
\[ R_2 = 4 \Omega \]

Find
\[ I_1, I_2, V_B, V_{R2} \]

Solution

\[ \text{KCL} \quad I_1 - I_0 - I_2 = 0 \]
\[ I_1 = I_0 + I_2 \]
\[ I_1 = 1 + I_2 \]
\[ I_1 = 1 - \frac{1}{10} \]
\[ \boxed{I_1 = 0.9 \text{ A}} \]

\[ V_B = V_A = I_2 R_2 = -0.4 \text{ V} \]

\[ V_{R2} = I_1 R_1 = 5.4 \text{ V} \]

\[ \text{KVL:} \quad V_s - V_{R1} - V_{R2} = 0 \]
\[ V_s = I_1 R_1 - I_2 R_2 = 0 \]
\[ V_s = (1 + I_2) R_1 - I_2 R_2 = 0 \]

\[ I_2 = \frac{-(V_s - R_1)}{-(R_1 + R_2)} \]
\[ I_2 = \frac{-(5 - 6)}{-(10 + 4)} = \frac{-1}{10} \text{ A} \]
\[ I_2 = -\frac{1}{10} \text{ A} \]
Finding Power

\[ I_3 = -I_2 = 0.1 \, \text{A} \]
\[ V_A = -0.4 \, \text{V} \]
\[ V_S = 5 \, \text{V} \]
\[ I_1 = 0.9 \, \text{A} \]

\[ P_{V_S} = V_S I = 5 \times 0.9 = 4.5 \, \text{W} \quad \text{supplied} \]

\[ P_{R_1} = V I = \frac{V^2}{R} = I^2 R_1 = (0.9)^2 \times 6 = 4.86 \, \text{W} \quad \text{absorbed} \]

\[ P_{R_2} = V I = (0.4)(0.1) = 0.04 \, \text{W} \quad \text{absorbed} \]

\[ P_{I_0} = V I = (0.4)(1) = 0.4 \, \text{W} \quad \text{supplied} \]

\[ \text{Supplied} = \text{Absorbed} \]
\[ 4.5 + 0.4 = 4.86 + 0.04 \]
\[ 4.9 = 4.9 \]

Conserved
EX

10 V

1 Ω

V1

2 Ω

V2

Is

S1

S2

S3

10 Ω

2A = Is

FIND V1, V2

② ALL BRANCH CURRENTS

③ POWER

SOLVE USING

KCL NODE VOLTAGE EQUATIONS

KVL LOOP CURRENT EQUATIONS

SUPERPOSITION

THEVENIN EQUIVALENT EQUATIONS
KCL @ $V_1$:

\[
\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0
\]

SOLVE SIMULTANEOUSLY:

\[
10V_1 - 100 + 2V_1 + 5V_1 - 5V_2 = 0
\]

\[
17V_1 - 5V_2 = 100
\]

\[
V_1 = \frac{100 + 5V_2}{17}
\]

@ $V_2$:

\[
\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0
\]

\[
5V_2 - 5V_1 + V_2 = 20
\]

\[
6V_2 - 5V_1 = 20
\]

\[
6V_2 - 5 \left( \frac{100 + 5V_2}{17} \right) = 20
\]

\[
102V_2 - 500 - 25V_2 = 340
\]

\[
77V_2 = 834
\]

\[
V_2 = \frac{834}{77} = 10.83
\]

\[
10(10.83) - 5V_1 = 20
\]

\[
V_1 = 9.0
\]
Set \( I_s = 0 \) (open)

\[
\begin{align*}
1 & \quad V_1 \quad V_2' \quad 10 \\
10 & \quad 5 & \quad 10 \quad 0
\end{align*}
\]

Resistor simplification

\[
\frac{60}{17} = 3.53
\]

\[
I = 2.21
\]

\[
V'_1 = 10 - 1(2.21) = 7.79
\]

\[
I'_s = \frac{7.79}{5} = 1.56
\]

\[
I'_2 = 2.21 - 1.56 = 0.652 \quad V'_2 = 6.52 \text{ V}
\]

\[
\begin{align*}
V''_1 = 4.4 \quad -I''_2 = 2 \times 1.56 \\
V''_2 = 4.4
\end{align*}
\]

\[
\begin{align*}
I''_{10} &= 0.44 \\
I''_2 &= 2 - 0.44 = 1.56
\end{align*}
\]

\[
V''_1 = 4.4 - (2 \times 1.56)
\]

\[
V''_2 = 0.88
\]

\[
V_s = 6.52 + 4.4 = 10.9
\]

\[
V_1 = 7.79 + 0.88 = 8.7
\]
Thevenin Equivalent

\[ V_{oc} = \frac{I_{sc}}{I_{sc}} \]

\[ V_{oc} \text{ found in superposition} = 6.52 \text{ V} \]

Finding \( I_{sc} \)

\[ I = 4.11 \text{ A} \]

\[ V_A = 10 - I(4.11) = 5.89 \text{ V} \]

\[ I_{sc} = \frac{5.89 \text{ V}}{2A} = 2.95 \text{ A} \]

\[ R_{TH} = \frac{V_{TH}}{I_{sc}} = 2.21 \Omega \]

\[ V_{TH} = V_{oc} = 6.52 \text{ V} \]

\[ V_2 = 6.52 + 2(2.21) \]

\[ V_2 = 10.9 \text{ V} \]
THEVENIN EQUIVALENT CIRCUIT

Any resistive circuit, no matter how complex, can be reduced to a series voltage and resistance, at any terminal in that circuit.

\[
V_X = I_{TH} \left( \frac{R_{TH} R_L}{R_{TH} + R_L} \right)
\]
Find all branch currents and node voltages.

Step 1) Label all branches and nodes

Branch currents: $I_3, I_6, I_4, I_5, I_2$

Find voltages at all nodes then use Ohm's law

$$I_2 = \frac{V_2 - V_4}{2R}$$

$$I_6 = \frac{V_2 - 0}{6}$$

$$V_1 = -12\,V$$

$$V_3 = 10 + V_4$$

KCL @ node 2

$$\sum I = 0$$

$$I_3 + I_5 = I_6 + I_2$$

$$I_3 = \frac{V_1 - V_2}{3}$$

$$I_5 = \frac{V_3 - V_2}{5}$$

$$\frac{V_1 - V_2}{3} + \frac{V_3 - V_2}{5} = \frac{V_2}{6} + \frac{V_2 - V_4}{2}$$

KCL @ node 4

$$I_2 + 3 = I_4 + I_5$$

$$I_4 = \frac{V_4 - 0}{4}$$

4 eqn. 4 unknowns can be solved simultaneously.
Find $V_{oc}$

\[ I_2 \quad 20V \]

\[ V_{I1} \quad 10 \quad V_{I2} \quad 6 \quad 10 \]

\[ 2 \quad V_2 \quad 0 \quad 0 \]

\[ 5A \quad 4 \]

KCL @ $V_1$

\[ I_2 + I_1 = 5 + I_3 \]

\[ \frac{V_1 - (V_1 - 20)}{10} + \frac{V_1 - 0}{2} = 5 + I_2 \]

\[ \frac{V_1}{2} = 5 \]

\[ V_1 = 10 \]

\[ V_2 = -10 \]

\[ V_{oc} = -10 \]
**Find** \[ I_{SC} \]

\[ V_2 = V_1 - 20 \]

\[ I_{SC} = \frac{V_2}{6} = \frac{V_1 - 20}{6} \]

KCL \[ I_1 + I_2 = I_3 + 5A \]

\[ V_1 = \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{10} = \]

KCL \[ V_2 \]

\[ I_2 = I_3 + I_{SC} \]

**R_{TH}**

\[ R_{TH} = \frac{V_{OC}}{I_{SC}} \]

\[ V_{OC} \]

\[ 15 \text{ ohms} \]
Finding T.E.C

\[ I_L = \frac{V_{TH}}{R_{TH} + R_L} \]

\[ V_L = 15 \times I_L \]
**Diodes**

- **Forward**
  \[ V_D = V_F + I_D R_D \]
  \[ I_D = \frac{25\text{mV}}{I_D} \]
- **Reverse**

**Example 1**

Given: \( V_F = 0.7 \text{V} \)

What do we know?

\[ I_D = I \]

**Reverse**

No path, \( I = I_D = 0 \)

\[ V_D = 5 \text{V} \]

**Example 2**

Diode Forward

\[ I_D' = \frac{5 - 0}{10} = 0.5 \text{A} \]

\[ \frac{r_D}{r_D} = \frac{25\text{mV}}{0.5\text{A}} = 50 \text{m}\Omega \]

\[ \frac{5 + 10}{10} = 0.7 \text{V} \]

\[ r_D = 50 \text{m}\Omega \]

\[ I = 0.7 \text{A} \]

\[ 4.3 = I(10.05) \]

\[ V_D = 17 + 4.28(0.05) \]

\[ V_D = 17.72 \text{V} \]

\[ I = 0.428 \text{A} \]
IDEAL OP-AMPS

$A_0 = \infty$
$I_+ = I_- = 0$
$V_+ = V_-$

Find: $\frac{V_{out}}{V_{in}}$

$i_1, i_2$ IF $V_{in} = 5V$

$R_1 = 1k\Omega$
$R_2 = 10k\Omega$

ASSUME IDEAL OP-AMPS

WHAT DO WE KNOW?

I) $V_- = V_+ \quad I_+ = I_- = 0$

II) $V_1 = 0$ (CONNECTED TO GROUND) $V_+ = 0$

III) $i_1 = \frac{V_{in} - V_+}{R_1}$. OHM'S LAW $= \frac{5mA}{R_1}$

IV) $i_1 \bullet i_2 \rightarrow I_+$ KCL @ node $V_+$

$i_1 = i_2 + I_+ \quad \sum i_{in} = \sum i_{out}$

$i_1 = i_2 = 5mA$

V) $\frac{R_2}{\overrightarrow{V_+}} \quad \frac{i_2}{\overrightarrow{U_{out}}}$

$i_1 = i_2 \quad -\frac{V_{out}}{R_2} = \frac{V_{in}}{R_1} \quad \sqrt{\frac{V_{out}}{V_{in}}} = \frac{R_2}{R_1}$
ASSUME IDEAL OP-AMP
FIND \( \frac{V_{out}}{V_{in}} \), \( i_1 \), \( i_2 \), \( i_3 \) IF \( V_{in} = 5 \text{ V} \)
\[ R_1 = 5 \text{ k\Omega} \]
\[ R_2 = 1 \text{ k\Omega} \]

WHAT DO WE KNOW?

I) \( V_+ = V_- \), \( I_+ = I_- = 0 \)

II) \( V_+ = V_{in} \), \( i_1 = I_+ \), \( i_2 = i_3 \) (KCL)
\[ V_- = V_+ = V_{in} \]

III)

\[ \frac{V_{out}}{R_1} = i_2 \quad \text{AND} \quad \frac{V_{out}}{R_2} = i_3 \]
\[ i_2 = i_3 \]
\[ i_3 = \frac{V_{in} - 0}{R_2} \]
\[ i_2 = \frac{V_{out} - V_{in}}{R_1} \]

\[ \frac{V_{in}}{R_2} = \frac{V_{out} - V_{in}}{R_1} \]

\[ \frac{R_1 \cdot V_{in} + V_{in}}{R_2} = V_{out} \]

\[ \frac{R_1}{R_2} = 1 + \left( \frac{\frac{V_{out}}{V_{in}} - \frac{R_1 + R_2}{R_1}}{R_2} \right) \]

VOLTAGE DIVIDER
\[ \frac{V_2}{V_1} = \frac{R_2}{R_1 + R_2} \]
\[ V_2 = V_{in} \]
\[ V_1 = V_{out} \]
Using Ideal Properties:

\[ I_3 = 0 \quad I_2 = I_4 \]
\[ I_4 = 0 \quad I_5 = I_6 \]

\[ V_3 = V_1 \]
\[ V_3 = V_0 \left( \frac{6}{5+6} \right) = V_1 \]

\[ V_{\text{in}} - V_1 = V_1 - V_0 \rightarrow \frac{V_{\text{in}} - \frac{6}{11} V_0}{2} = \frac{6}{11} V_0 - V_0 \]

\[ 2 \frac{2}{11} V_0 = \frac{-5}{11} V_0 \]
\[ 2 V_{\text{in}} = \frac{7}{11} V_0 \]

\[ \frac{V_0}{V_{\text{in}}} = \frac{2.2}{7} \]
\[ V(t) = L \frac{d i(t)}{dt} \]

\[ L \rightarrow j \omega L = Z_L \]

\[ V = L j \omega I \]

\[ V_{\text{f.r.o.r}} = L j \omega I_m (\text{eddy}) \]

\[ Z_c = \frac{1}{j \omega C} \]

\[ V = j \omega I_m L \]

\[ V = (j \omega L) I_m \]
\[ V(t) = R \frac{d}{dt} [e^{lt}] \]

\[ V(t) = 10 \cos (\omega t + \theta) \]

\[ V(t) = 10 \cos (\omega t + \theta) \]

\[ V = I R \]

\[ I = \text{Im} [V(t)] \]

\[ V = I R \]

DC

\[ \frac{d}{dt} [e^{lt}] = L \cdot \text{Im} [V(t)] \]

\[ \frac{d}{dt} [e^{lt}] = C \cdot \text{Im} [V(t)] \]

AC circuits

Apparatus

Instructions

Resistors
\[ I(t) = 459 \cos(10t - 62.6^\circ) \, A \]

\[ I = \frac{459}{92.6^\circ} \]

\[ \frac{5.79 + 0.997}{5.79 + 5} = \frac{5.79}{5} \]

\[ 0.10 + \frac{3}{4} \times \frac{5}{5.79} = I \]

\[ \frac{3}{4} = \frac{1}{10} \times 1 \]

\[ V = I \times 2 \]

\[ F = 1 \times 3 F \]

\[ S \cos(10t) \]
**Transient Circuits**

Find $V_c(t)$, $i_1(t)$

---

**What do we know?**

- $i_c = C \frac{dV_c(t)}{dt}$ \hspace{1cm} **U-I Law of Capacitors**
- $i_c = i_1$ \hspace{1cm} **KCL**
- $\dot{i}_1 = \frac{V_{in} - V_c(t)}{R_1}$ \hspace{1cm} **Ohms Law**

---

I) $\frac{V_{in} - V_c(t)}{R_1} = C \frac{dV_c(t)}{dt}$

\[ \frac{dV_c(t)}{dt} + \frac{1}{RC} V_c(t) = \frac{1}{RC} V_{in}(t) \] \hspace{1cm} **First order, constant coeff. Diff. Eq.**
If \( u_{in}(t) = A e^{3t} \) \( u_c(t) = Ke^{5t} \)

**Homogeneous Solution**

\[
\frac{d}{dt} u_c(t) + \frac{1}{RC} u_c(t) = 0
\]

\[K e^{5t} + \frac{K}{RC} e^{5t} = 0
\]

\[K e^{5t} \left( s + \frac{1}{RC} \right) = 0 \quad \text{[} s = -\frac{1}{RC} \text{]} \quad u_c(t) = Ke^{-\frac{t}{RC}}
\]

**Particular Solution**

\[u_c(t) = V_c e^{3t}
\]

\[3 V_c e^{3t} + \frac{V_c}{RC} e^{3t} = A e^{3t}
\]

\[3 V_c + \frac{V_c}{RC} = A
\]

\[V_c = \frac{A}{3 + \frac{1}{RC}} \quad \text{[} u_c(t) = \frac{A}{3 + \frac{1}{RC}} e^{3t} \text{]}
\]

**Total**

\[u_c(t) = Ke^{-\frac{t}{RC}} + \frac{A}{3 + \frac{1}{RC}} e^{3t} \quad \text{[} t > 0 \text{]}
\]

Assume \( u_c(t) = 0 \) for \( t < 0 \) (Initial Conditions)

\[0 = K + \frac{A}{3 + \frac{1}{RC}} \quad K = -\frac{A}{3 + \frac{1}{RC}}
\]

\[u_c(t) = \frac{A}{3 + \frac{1}{RC}} \left[ e^{3t} - e^{-\frac{t}{RC}} \right] \quad \text{[} t > 0 \text{]}
\]

As \( t \to \infty \) steady state \( u_c(t) \to \frac{A}{3 + \frac{1}{RC}} e^{3t} \)
\[ v_c(t) = \frac{A}{3 + \frac{1}{RC}} \left[ e^{3t} - e^{-\frac{t}{RC}} \right], \quad t > 0 \]

\[ R = 1 \, k\Omega \]
\[ C = 1 \, \mu F \]
\[ v_{in}(t) = 5 \, e^{3t} \]

\text{Steady State Expression}

As \( t \to \infty \)
\[ v_c(t) \to \left( \frac{A}{3 + \frac{1}{RC}} \right) e^{3t} \]
\[ \frac{dx(t)}{dt} + ax(t) = f(t) \]

If \( x_0(t) \) is a soln, and \( x_c(t) \) is a soln to the homogeneous eqn:

\[ \frac{dx(t)}{dt} + ax(t) = 0 \]

Then

\[ x(t) = x_0(t) + x_c(t) \]

Let \( f(t) = A \) constant

\[ \frac{dx_0(t)}{dt} + ax_0(t) = A \quad x_0(t) = k_1 \]

\[ \frac{dx_c(t)}{dt} + ax_c(t) = 0 \quad x_c(t) = k_2 e^{-at} \]

\[ x(t) = k_1 + k_2 e^{-at} \]
Using Laplace Transforms

\[ V_c(t) + CR \frac{dV_c}{dt} = 0 \quad V(0^-) = V_s \]

\[ V(s) + CR(sV(s) - V(0^-)) = 0 \]

\[ V(s)(1 + CRs) - CRV(0^-) = 0 \]

\[ V(s) = \frac{CRV_c}{1 + CRs} = \frac{V_s}{RC + s} \]

\[ u(t) = V_s e^{-\frac{t}{RC}} \]

\[ \frac{1}{s+a} \Rightarrow e^{-at} \]

\[ \frac{1}{s} \Rightarrow \frac{1}{s} \]

\[ \frac{dV_c(t)}{dt} + \frac{1}{RC} V_c(t) = \frac{V_s}{RC} \]

\[ sV(s) + \frac{V(s)}{RC} = \frac{V_s}{SRC} \]

\[ V(s)(s + \frac{1}{RC}) = \frac{V_s}{RC} \frac{1}{s(s + \frac{1}{RC})} = \frac{\frac{V_s}{RC}}{s} + \frac{\frac{V_s}{RC}}{s + \frac{1}{RC}} \]

\[ \frac{V_s}{RC} \]

\[ \frac{V_s}{RC} \]

\[ \frac{1}{s + \frac{1}{RC}} \]

\[ s = -\frac{1}{RC} \]

\[ u(t) = V_s - V_s e^{-\frac{t}{RC}} \]

AC/SS CRT Analysis

INIT. Cond. = 0
SINUSOIDAL RESPONSE

\[ V_s(t) = V_m \cos(\omega t + \phi) \]

\[ L \frac{di(t)}{dt} + Ri(t) = V_m \cos(\omega t + \phi) \]

\[ i(t) = -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi_v - \theta) e^{-(\frac{R}{L})t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi_v - \theta) \]

 homogeneous (transient)

as \( t \to \infty \)

as terms \( \to 0 \)

\[ i_{ss}(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi_v - \theta) \]

\[ I_{\text{max}} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \]

Let \( \phi_v - \theta = \theta_i \)

\[ i_{ss}(t) = I_m \cos(\omega t + \theta_i) \]
AC CIRCUIT ANALYSIS - STEADY STATE ANALYSIS

CONVERT TO PHASOR DOMAIN

\[ v(t) = A\cos(\omega t + \theta) = \text{Re} \left[ A\ e^{j(\omega t + \theta)} \right] \]

ALSO WRITTEN AS A PHASOR \[ \mathbf{V} = A\ \mathbf{\Phi} \]

CONVERT COMPONENTS

<table>
<thead>
<tr>
<th>TIME DOMAIN</th>
<th>PHASOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>( R )</td>
</tr>
<tr>
<td>( L )</td>
<td>( j\omega L )</td>
</tr>
<tr>
<td>( C )</td>
<td>( \frac{1}{j\omega C} )</td>
</tr>
</tbody>
</table>

\[ 5\cos(10t) \quad \rightarrow \quad \frac{5}{\sqrt{10}} \quad \rightarrow \quad 5\ \angle 40^\circ \]

\[ I = \frac{5\ \angle 40^\circ}{5 - \frac{j}{3} + j10} \]

\[ I = \frac{5\ \angle 40^\circ}{5 + j(10 - \frac{1}{3})} \]
**Phasor Notation** — A complex number that carries amplitude and phase angle of a sinusoidal function

\[ e^{j\phi} = \cos \phi + j \sin \phi \]

\[ \text{Re} \{e^{j\phi}\} = \cos \phi \]
\[ \text{Im} \{e^{j\phi}\} = \sin \phi \]

\[ v(t) = V_m \cos (wt + \phi) \]
\[ = V_m \text{Re} \{e^{j(wt + \phi)}\} = V_m \text{Re} \{e^{jwt} e^{j\phi}\} \]
\[ = \text{Re} \{V_m e^{j\phi} e^{jwt}\} \]

**Phasor of** \( v(t) \)

\[ \vec{V} = V_m e^{j\phi} = V_m \cos \phi + j V_m \sin \phi = V_m \angle \phi \]

**Example**

\[ \vec{V} = 100 \angle -26^\circ \rightarrow v(t) = 100 \cos(wt - 26^\circ) \]
\[ i(t) \]

\[ v(t) = L \frac{di(t)}{dt} \]

\[ v = V_m e^{\theta_v} e^{jwt} \]

\[ V_m e^{\theta_v} e^{jwt} = L \int I_m e^{\theta_i} dwe^{jwt} \]

\[ V_m e^{\theta_v} = I_m e^{\theta_i} jwc \]

\[ \vec{V} = \int I \, dwc \]

\[ Z_L \text{Inductor Impedance is now complex and frequency dependent} \]

\[ i_c = C \frac{dv_c(t)}{dt} \]

\[ I_m e^{\theta_i} e^{jwt} = C V_m e^{\theta_v} dwe^{jwt} \]

\[ I_m e^{\theta_i} = jwc V_m e^{\theta_v} \]

\[ V_m e^{\theta_v} \]

\[ \int I \text{ } Z_c \]

\[ Z_c = \frac{1}{jwc} \]
\[ I = \frac{5 \angle 0^\circ}{5 + j9.67} = \frac{5 \angle 0^\circ}{10.89 \angle 62.6^\circ} = 0.459 \angle -62.6^\circ \]

\[ i(t) = 0.459 \cos(10t - 62.6^\circ) \]

**Summary of Complex Numbers**

\[ x + jy = re^{\theta} \]

\[ r = \sqrt{x^2 + y^2} \]

\[ \theta = \tan^{-1} \frac{y}{x} \]

\[ x = r \cos \theta \]

\[ y = r \sin \theta \]

\[ \frac{1}{e^{\theta}} = e^{-\theta} \quad \text{or} \quad \frac{1}{5 \angle 30^\circ} = 0.2 \angle -30^\circ \]

**Complex Impedance**

\[ Z = R + jX \]

\[ \begin{array}{c}
\text{Real} \\
\text{reactive}
\end{array} \]

\[ V = I Z \]
MAGNETIC FIELDS

\[ H = \frac{B}{\mu} = \frac{I}{\mu} \frac{\partial \Phi}{\partial t} \]

- \( H \) = Magnetic Field Strength (A/m)
- \( B \) = Magnetic Flux Density (Tesla)
- \( \Phi \) = Magnetic Flux
- \( I \) = Current
- \( \mu \) = Permeability of medium
  - Air: \( 4\pi \times 10^{-7} \) H/m

\[ \Phi = \int B \cdot dS = \text{Total Flux in a Volume} \]

\[ V = -\frac{N \partial \Phi}{dt} \]

Induced Voltage

Total Volume of Material Density \( \rho \) is Weight

Total Volume of \( B \) is \( \Phi \)
THE IDEAL TRANSFORMER

\[ \frac{V_1}{N_1} = \frac{V_2}{N_2} \]

\[ N_1 I_1 = -N_2 I_2 \]

\[ a = \frac{N_2}{N_1} \]

\[ \frac{V_1}{N_1} = -\frac{V_2}{N_2} \]

\[ N_1 I_1 = N_2 I_2 \]

\[ I_2 = \frac{I_1}{a} \]

\[ Z_{in} = \frac{Z_L}{a^2} \]

coil 1: \( P_1 = V_1 I_1 \), absorbed

coil 2: \( P_2 = V_2 I_2 \), supplied.
IDEAL TRANSFORMERS

\[ \frac{U_1}{U_2} = \frac{N_1}{N_2} \quad \frac{I_1}{I_2} = -\frac{N_2}{N_1} \]

\[ V_1 I_1 + V_2 I_2 = 0 \quad \text{NO POWER LOSS IN AN IDEAL XFORMER} \]

\[ n = \frac{N_2}{N_1} \quad \text{:= turns ratio} \]

\[ V_1 = \frac{V_2}{n} \quad I_1 = n I_2 \quad Z_1 = \frac{Z_2}{n^2} \]
IDEAL TRANSFORMERS

\[ \frac{V_1}{V_2} = \frac{N_1}{N_2} \]
\[ \frac{I_1}{I_2} = -\frac{N_2}{N_1} \]

\[ V_g = 2500 \cos 400t \]

**Find steady state** \( I_1, I_2, V_1, V_2 \)

\[ V_g \rightarrow V_a = 2500 \angle 0^\circ \]

\[ L_1 = 5 \text{ mH} \]
\[ Z_{L_1} = j(5 \text{ mH})(400) = j2 \]

\[ L_2 = 125 \text{ mH} \]
\[ Z_{L_2} = j(125 \text{ mH})(400) = j0.05 \]

\[ \frac{V_g - V_1}{2.5 + j2} = \pm 1 \]

\[ \frac{I_1}{-I_2} = -\frac{N_2}{N_1} \]
\[ \frac{I_1}{I_2} = \frac{1}{10} \]
\[ 10I_1 = I_2 \]

**Steady state \( \rightarrow \) phasors**

\[ \mathbf{V} = V \angle \theta \]

\[ Z_L = j \omega L \]
\[ Z_C = \frac{1}{j \omega C} \]

\[ V_2 = I_2 (2375 + j0.05) \]

\[ V_2 = \frac{V_1}{10} \]

\[ V_1 = 10I_2 (2375 + j0.05) \]

\[ V_1 = (10)(10)I_1 (2375 + j0.05) \]

\[ V_1 = I_1 (23.75 + j5) \]
\[ V_{6a} = I_1 \left( .25 + j2 \right) + V_i \]
\[ = I_1 \left( .25 + j2 \right) + I_1 \left( 23.75 + j5 \right) \]
\[ V_{6a} = I_1 \left( 24 + j7 \right) \]

\[ 25 \angle 0^\circ = I_1 \]
\[ \frac{25 \angle 60^\circ}{2.5 \angle 26^\circ} \]

\[ I_2 = 10 \times I_1 = 1000 \cos \left( 400t - 16.26^\circ \right) \text{ A} = V_2(t) \]

\[ V_i = I_1 \left( 23.75 + j5 \right) = \left( 100 \angle -16.26^\circ \right) \left( 24.27 \angle 11.89^\circ \right) \]
\[ = 242.7 \angle -4.37^\circ \]

\[ V_i(t) = 242.7 \cos \left( 400t - 4.37^\circ \right) \text{ V} \]

\[ V_2 = \frac{V_i}{10} = 242.7 \angle -4.37^\circ \]

\[ V_2(t) = 242.7 \cos \left( 400t - 4.37^\circ \right) \text{ V} \]
\[ V_1 = \frac{V_2}{n} \quad I_1 = -n I_2 \quad n = \frac{1}{4} \]

\[ Z_1 = \frac{Z_L}{n^2} = 4^2 Z_L \]

\[ I_1 = \frac{120 \degree}{18 - 4^4 + 32 + 1} = 2.33 \angle -13.5^\circ \]

\[ V_1 = I_1 Z_1 = (2.33 \angle -13.5^\circ)(32 + j16) \]

\[ = 83.49 \angle 13.07^\circ \]

\[ V_2 = -V_1 n = \frac{1}{4} 83.49 \angle 13.07^\circ = 20.87 \angle 193.07^\circ \]

\[ I_2 = -\frac{I_1}{n} = 4 \left( 2.33 \angle 13.5^\circ \right) \approx 9.33 \angle 166.50^\circ \]
**DC**

\[ P = \text{Power} \]
\[ i = \text{Current} \]
\[ V = \text{Voltage} \]

Positive Power (absorbed)

\[ P_s = -VI \]
\[ P_R = \frac{V}{R}I = \frac{I^2}{R} = \frac{V_R^2}{R} \]
\[ V = IR \]

\[ P_{R2} = V_{R2}I = \frac{V_{R2}^2}{R_2} \]

**AC**

\[ V(t) = V_m \cos(\omega t + \theta_v) \]
\[ i(t) = I_m \cos(\omega t + \theta_i) \]

Using an arbitrary reference time we can write

\[ i(t) = I_m \cos \omega t \]
\[ V(t) = V_m \cos(\omega t + \theta_v - \theta_i) \]

\[ P = V(t)i(t) = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t \]
\[
\text{USING: } \cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta)
\]
\[
P(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2wt + \theta_v - \theta_i)
\]
\[
= \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos((\theta_v - \theta_i) \cos 2wt
\]
\[
- \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin 2wt
\]

**AVERAGE (REAL) POWER**

\[
P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)
\]

**REACTIVE POWER**

\[
Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)
\]

\[
p(t) = P + P \cos 2wt - Q \sin 2wt
\]

*P is not time dependent. (Constant, like DC)*

\[
P = (V_{\text{rms}}) (I_{\text{rms}}) (pf)
\]

*V_{\text{rms}} \text{ DC Equivalent Root Mean Squared}*

*I_{\text{rms}} *

For sinusoidal \( V(t), i(t) \)

\[
V_{\text{rms}} = \frac{V_m}{\sqrt{2}}, \quad I_{\text{rms}} = \frac{I_m}{\sqrt{2}}
\]

*pf: power factor \cos(\theta_v - \theta_i)*
\[-1 < pf < 1 \quad pf > 0 \quad \text{lagging } pf \quad \text{current lags voltage}\]

\[pf < 0 \quad \text{leading } pf \quad \text{current leads voltage}\]

\[\text{EX: } \begin{array}{c}
\begin{array}{c}
\text{Current} \\
\text{Voltage}
\end{array} \\
\begin{array}{c}
\text{1} \\
\sqrt{2}
\end{array}
\end{array} \]

\[V = 100 \cos(wt + 15^\circ) \quad V \]

\[i = 4 \sin(wt - 15^\circ) \quad A\]

\[\text{Find active power and reactive power at terminals}\]

\[V = 100 / 15 \quad I = 4 / -15 - 90 = 4 / -105\]

\[P = \frac{100(4) \cos(15 + 105^\circ)}{2} = 200(-.5) = -100 \quad \text{W} \quad \text{supplied}\]

\[Q = \frac{100(4) \sin(120^\circ)}{2} = +173.2 \quad \text{VAR}\]
0) \[ v = 100 \cos(\omega t - 45^\circ) \]
\[ i = 20 \cos(\omega t + 15^\circ) \]
\[ P = \frac{100 \times 20}{2} \cos(-45 - 15) \]
\[ = 100 \times 5 = 500 \text{ W} \]

Power from A to B

\[ Q = \frac{100 \times 20}{2} \sin(-60) \]
\[ = -866.03 \text{ VAR} \]

Delivering (Magnetic) vars from B to A

\[ \rho(t) = 50 + 50 \cos 2\omega t + 866.03 \sin 2\omega t \]

Energy being stored in magnetic fields associated with inductive elements.
\[ V(t) = V_m \cos(\omega t + \theta_v) \]
\[ I(t) = I_m \cos(\omega t + \theta_i) \]

**Instantaneous Power**
\[ p(t) = V(t) I(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i) \]

\[ p(t) = \frac{V_m I_m}{2} \left[ \cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i) \right] \]

\[ \uparrow \quad \text{TIME IND.} \quad \uparrow \quad \text{TIME DEP.} \]

**Average Power**
\[ P_{av} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \frac{V_m I_m}{\sqrt{2}} \frac{\cos(\theta_v - \theta_i)}{\sqrt{2}} \]

\[ \rho_f \equiv \text{power factor} = \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \rho_f \]

\[ V_{rms} I_{rms} \equiv \text{apparent power} \]

\[ 1 \leq \cos(\theta_v - \theta_i) \leq 1 \]

0 < \rho_f < 1 \quad \text{lagging} \quad \rho_f \]

-1 < \rho_f < 0 \quad \text{leading} \quad \rho_f \]

\[ V = 10 \, \text{V} \]
\[ I = \frac{10 \, \text{A}}{\sqrt{2}} \approx 7.07 \, \text{A} \]

\[ P_{av} = \frac{10 \times (7.07)}{2} \cos(60^\circ - 15^\circ) = 12.48 \, \text{Watts} \]

\[ \rho_f = \cos(45^\circ) = 0.707 \quad \text{lagging} \quad \text{(inductive load)} \]
**Complex Power**

\[ S = P + jQ \]

\[ P = \text{Re}\{S\} \quad \text{Watts} \]

\[ Q = \text{Im}\{S\} \quad \text{Vars} \]

\[ S \quad \text{Volt-Amps} \]

\[ \theta = \theta_u - \theta_i \]

\[ |S| = \sqrt{P^2 + Q^2} \quad \tan \theta = \frac{Q}{P} \]

\[ |S| \quad \text{Apparent Power} \quad \text{Volt-Amps} \]

**EXAMPLE**

Load @ 240 V \(_{\text{rms}}\) absorbs 8 kW w/ pf 0.8 lag

a) Find \( S \)

b) Find Impedance of the load

**Solution**

\[ P = |S| \cos \theta \]

\[ \cos \theta = 0.8 \quad \theta = 36.87^\circ \]

\[ \sin \theta = 0.6 \]

\[ |S| = 10 \text{ KVA} \]

\[ Q = |S| \sin \theta = 10 \text{ KVAR (0.6)} = 6 \text{ KVAR} \]

\[ S = (8 + j6) \text{ KVA} \]
\[ P = V_{rms} I_{rms} \cos(\theta_j - \theta_i) \]

\[ 8000 = 240 V \ I_{rms} (0.8) \]

\[ I_{rms} = \frac{8000}{240(0.8)} = 41.67 \text{ Amps} \]

\[ |Z| = \left| \frac{V_{rms}}{I_{rms}} \right| = \frac{240}{41.67} = 5.76 \]

\[ \Delta \theta = \Delta \theta_j - \Delta \theta_i = 36.87 \]

\[ Z = 5.76 \angle 36.87 = 4.608 + j3.456 \quad \omega = Z \]
Industrial load user 88 kW @ pf = 0.90 lag.
From 480 V rms line.

a) Find power needed from power co.

\[ P_L = 88 \text{ kW} = I_{\text{rms}} V_{\text{rms}} (pf) \]
\[ 88 \text{ kW} = I_{\text{rms}} (480) (0.707) \]
\[ I_{\text{rms}} = 259.3 \text{ A} \]

\[ P_L = I_{\text{rms}}^2 (R) + 88 \text{ kW} \]
\[ P_L = 93.38 \text{ kW} \]

b) \( pf = 0.90 \)
\[ I_{\text{rms}} = 203.7 \text{ A} \]
\[ P_L = 91.22 \text{ kW} \]

Complex power.

\[ S = V_{\text{rms}} I_{\text{rms}}^* = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) + j V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i) \]

Real power

\[ S = P + j Q \]

\[ P = \text{Re}(S) = V_{\text{rms}} I_{\text{rms}} \]
Frequency CONTENT of a SIGNAL

How fast sample?

Bandwidth

Filter

System

Range of frequencies

Voice

60 → 12 kHz

Output

$Y(t)$

$X(t)$

$\omega_2$

$\omega_1$

$\omega_0$
Resonant Circuits

\[ Z_L = \frac{1}{j\omega L} \]

\[ Z_C = \frac{1}{j\omega C} = \frac{1}{sC} \]

\[ Z_0 = Z_L = Z_C \]

\[ V \]

\[ I \]

\[ V_1 \]

\[ V_2 \]

\[ V_3 \]

High Pass Filter (HPF)

Low Pass Filter (LPF)
The image contains handwritten notes and diagrams. The text appears to be related to electrical engineering or a similar technical field. The diagrams show various electrical circuits and waveforms. The handwritten text includes terms like "amplifier," "gain," and "frequency transfer function." There are also notes about signals and waveforms, suggesting a discussion on signal processing or electronic circuit analysis.
SERIES RESONANCE C4.15

\[ Z_T = R + j \omega L - \frac{j}{\omega C} \]
\[ = R + j \left( \omega L - \frac{1}{\omega C} \right) \]
\[ |V_L| = |V_H| \text{ BUT ARE } 180^\circ \text{ OUT OF PHASE} \]

\( \omega L = \frac{1}{\omega C} \)
\( \omega_0^2 = \frac{1}{L C} \)
\( \omega_0 = \frac{1}{\sqrt{L C}} \)
\( f_0 = \frac{1}{2\pi \sqrt{LC}} \)

\[ Z_T = R \quad |Z_T| \text{ IS MINIMUM} \]
\[ I \text{ IS MAXIMUM} \]

\[ Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} \quad \text{Quality Factor} \]

\[ B_W = \frac{\omega_0}{Q} \quad \text{BANDWIDTH} = f_0 - f_L \]
\[ V = 10 \angle 0 \] 

\[ N \] 

\[ 2 \] 

\[ 10 \mu F \] 

\[ 25 mH \] 

\[ I \] 

\[ \omega_0 = \frac{1}{\sqrt{LC}} = 2000 \] 

@ \( \omega_0 \), \( I = \frac{V}{R} \) \( \frac{B}{C} \) \( d\omega L - \frac{1}{d\omega C} = 0 \) 

\( = 5 \angle 0^\circ \) 

\( V_R = I(2) = 10 \angle 0^\circ \) 

\( V_L = j\omega L I = 250 \angle 90^\circ \) 

\( V_C = \frac{1}{j\omega C} (I) = 250 \angle -90^\circ \) 

\( Q = \frac{\omega_0 L}{R} = 25 \) 

\[ \text{Notice: } \quad |V_L| = \omega_0 L |I| = \frac{\omega_0 L}{R} |V_S| = Q |V_S| \] 

\[ |V_C| = \frac{1}{\omega_0 C} |I| = \frac{1}{\omega_0 C R} |V_S| = Q |V_S| \]
\[ V_c = \frac{V}{R} \left( \frac{1}{\omega L} \right) = 2000 \]

\[ Z = \frac{1587}{1.783} = 900 \]

\[ Z_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} \]

\[ I = \frac{Z_0}{R} = \frac{15.87}{0.63} = 25.3 \quad \text{mA} \]

\[ f = \frac{1}{2\pi \sqrt{L C}} \]

\[ C = 1277 \, \text{F} \]

\[ L = 0.02 \, \text{H} \]

Choose \( C \) and \( V_c \) (constant)
Parallel Resonance

\[ Y_T = \frac{1}{R} + \frac{1}{\omega L} + j\omega C \]

\[ Y_T = \frac{1}{R} + j(\omega C - \frac{1}{\omega L}) \]

@ \( \omega C = \frac{1}{\omega L} \)

\[ \omega_a^2 = \frac{1}{LC} \]

\[ \omega_p = \frac{1}{\sqrt{LC}} \]

\[ f_0 = \frac{1}{2\pi \sqrt{LC}} \Rightarrow Y_T \text{ is minimum} \]

\[ Z_T \text{ is maximum} \]

\[ V_{out} \text{ is maximum} \]

\[ Q = \frac{R}{\omega_a L} = \frac{\omega_a}{R} \]

\[ Bw = \frac{\omega_a}{Q} \]

\[ Bw = f_0 - f_l \]
$V_s = 120 \angle 0^\circ \quad R = 100 \quad C = 600 \mu F \quad L = 120 \, mH$

Find all $I$, $V_o$ @ resonance.

\[ \omega_0 = \frac{1}{\sqrt{LC}} = 117.85 \, rad/s \]
\[ Y_C = j\omega_0 C = j70.7 \times 10^{-3} \]
\[ Y_L = \frac{1}{j\omega_0 L} = -j70.7 \times 10^{-3} \]

\[ I_R = \frac{V_s}{R} = \frac{120 \angle 0^\circ}{100} = 1.2 \angle 0^\circ \]
\[ I_C = V_s Y_C = 8.49 \angle 90^\circ \]
\[ I_L = V_s Y_L = 8.49 \angle -90^\circ \]

\[ I_S = I_R + I_C + I_L = 1.2 \angle 0^\circ = I_S \]

\[ V_o = V_S \quad (or \, of \, course) \]